Chapter 3. Demographic model of humanity

As it was shown above, growth of population is one of the main factors featuring humanity as a system so it is reasonable to consider in more details its characteristics in the context of the dimension and similarity analysis and system analysis as well.

3.1. Population growth model

Chapter 1 identified a range of demographic transition models [77, 78, 79, 80]. The most recent of them proposed by Korotaev A.V. et al accounts for developments of other authors and is based on the idea that the demographic transition is caused by higher female literacy and statistics prove this to a certain extent.

However, the dynamics of the global aggregate fertility rate (AFR)[81] (fig. 3.1) indicates AFR being constantly high up to 1970 and the female literacy level did not affect it.

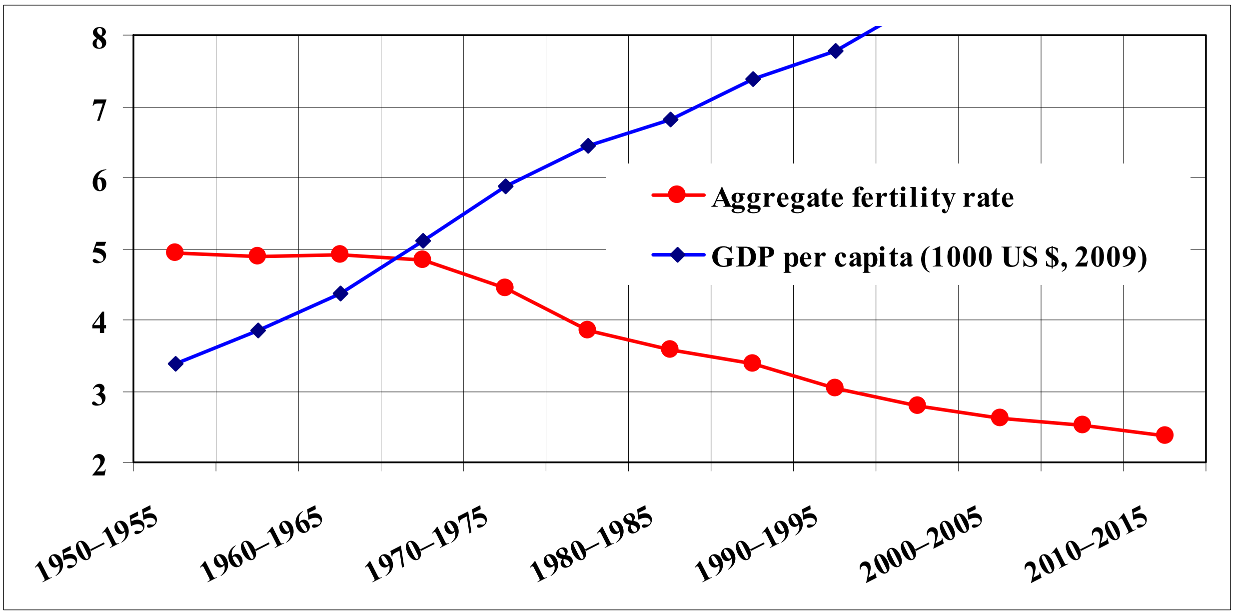

Figure 3.1.Dynamics of global aggregate fertility rate (AFR)

In 1970, global population numbered around 3.7 billion and the literacy level was above 50 percent (fig. 3.2). Though the female literacy fell a bit behind the average level[82], it was quite high and should the female literacy be the principal reason of the demographic transition, AFR would have started decreasing gradually before 1970.

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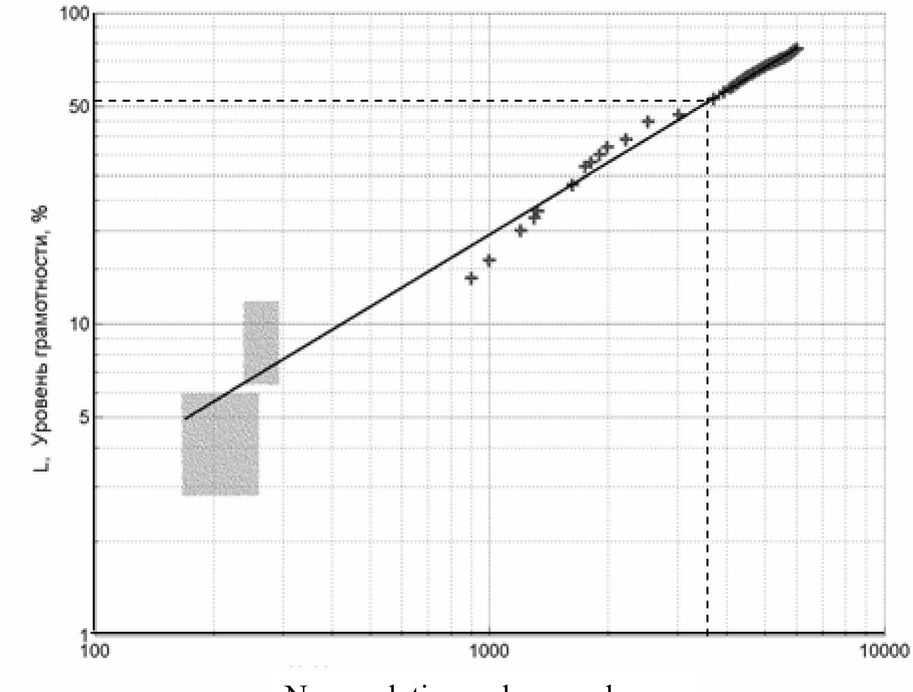
77 Капица С.П. Математическая модель роста населения мира. – М., 1992.
78 Kremer, M. Population Growth and Technological Change: One Million B.C. to 1990. 1993.
79 Подлазов А.В. Основное уравнение теоретической демографии. – М., 2001.
80 Коротаев А.В., Малков А.С., Халтурина Д.А. Математическая модель роста населения Земли, экономики, технологии и образования. – М., 2005.
81 Динамика СКР в мире (1950–2015). Прогноз ООН от 2010 года, средний вариант. – Википедия, 2013. https://ru.wikipedia.org/wiki
82 Системный мониторинг. Глобальное и региональное развитие / Отв. ред.: Д.А. Халтурина, А.В. Коротаев. – М., 2010. – С. 18.

L, Literacy level, %



N, population, mln. people

Figure 3.2. Literacy level versus global population[83]

But the authors excluded GDP per capita from the parameters that could decrease the fertility rate. They did so because they took as a contradiction ‘a low fertility rate in Russia and other former East European Soviet states and the dramatic fall in their life standards’[84].

However this contradiction may be illusory since the complexity theory[85] [86] and physics suggest such a phenomenon as hysteresis that means the direct and backward processes evolve differently.

Meanwhile, industrialization occurring in many countries somewhat alongside with the demographic transition is known to be accompanied not just by higher literacy level but by higher female employment level as well. To combine employment and parenting (hereafter parenting means the full cycle of activities covering childbirth, nursing, all-around provision and fostering) is challenging. So the number of children in families falls to the level that allows combining employment and parenting whereas the increasing female literacy is a subordinate process since being employed implies higher qualification and higher literacy accordingly. This hypothesis suggests that the higher female literacy is not a reason but is a consequence of a more significant, from the economics perspective, process of increasing employment level among women. Noting this hypothesis, Kremer’s model seems the most coherent though too sophisticated.

M. Kremer’s principal idea[87] which I will use suggests the fertility rate depending on the GDP per capita (see fig. 1.5). Figure 3.3 represents the correlation between the aggregate fertility

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[83] Cited from: Коротаев А.В., Малков А.С., Халтурина Д.А. Математическая модель роста населения Земли, экономики, технологии и образования. – М., 2005.

[84] ibid.

[85] Стэплтон Т. Маркетинг в условиях сложности: Учеб. пособие. – Кн.1 /Пер. с англ. – Жуковский, 2006.

[86] Oliva, T.A., Oliver, R.L. and McMillan, I.C. (1992) “A catastrophe model for developing service satisfaction”, *Journal of Marceting,* Vl.56, July, pp.83-95.

[87] Kremer, M. Population Growth and Technological Change: One Million B.C. to 1990.

rate and the GDP per capita (in PPP terms) for 85 countries of population above ~ 10 mln[88]. Despite its random pattern, the correlation proves evidently to be like the higher GDP per capita the lower fertility rate. Yet at G/N > 7000 dollars (dollars as per 2009) the aggregate fertility rate drops to 2–3, i.e. to the level when the population size changes relatively slow.

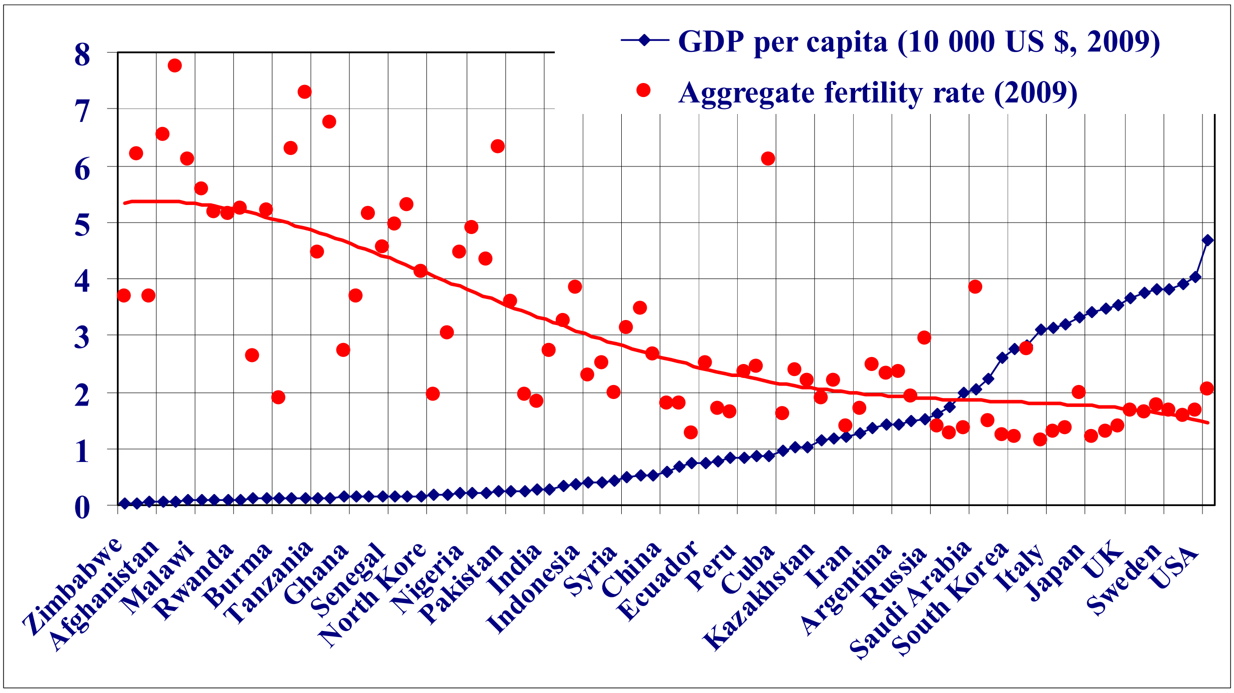

Figure 3.3 Correlation between fertility rate and GDP per capita

The next logical assumption is that it is a certain economic logic rather than just an unwillingness of wealthy families to bear many children underpins the demographic transition. Women and families are likely to choose between two alternatives (just parenting or employment and less number of children). If employment provides income above a definite level corresponding to the value of childbirth, a woman would prefer employment over parenting more children. And the higher income her employment offers a woman the slenderer she is inclined to parenting.

There is no denying the role of cultural factors but economic reasons are surely stronger. If a man is unable to provide for a family of five children and two adults, then two employees and two children will be a more attractive alternative which allows to avoid a bare subsistence life style.

It is a logic assumption in the mathematical model of the global population growth that the population increase **dN** over the period **dT** is proportional to three factors:

1) number of people – **N**;

2) amount of excessive GDP per capita – **(G/N–m)** that provides for childbirth and parenting (m – a subsistence level which provides a zero reproduction rate, see fig. 1.11);

3) a certain constrain factor which features characteristics of the abovementioned choice against the alternative cost principle and covers:

- increasing fertility ratio at low **G/N,**
- decreasing fertility ratio at high **G/N**.

[88] The World Factbook. CIA, 2009. https://commons.wikimedia.org/wiki/File:TFR_vs_PPP_2009.svg?uselang=ru

To make the model more analytic than M. Kremer's is, let's choose the constrain factor of the simplest type that is structurally similar to the factor from the common logistic growth equation which represents reproduction of sub-human life forms. So the constrain factor is 1 – k•G/N, where k is a constant. Hence the differential population growth equation takes the following general form:

$$dN/dT = A \cdot N \cdot (G/N - m) \cdot (1 - k \cdot G/N). \qquad (3.1)$$

To determine variable **G/N**, let's use the equation (1.11) defined earlier

$$G = N \cdot (m + \gamma N).$$

So equation (3.1) may be transformed to

$$dN/dT = A \cdot \gamma \cdot (1 - k \cdot m) \cdot N^2 \cdot (1 - k \cdot \gamma \cdot N/(1 - k \cdot m)). \qquad (3.2)$$

And then it may be simplified to

$$dN/dT = (1/C) \cdot N^2 \cdot (1 - N/N_{max}). \qquad (3.3)$$

When $N/N_{max} \to 0$, equation (3.3) transforms to the equation of (1.2) type which represents a hyperbolic population growth. When $N/N_{max} \to 1$, equation (3.3) transforms to the equation dN/dT = 0 with the solution $N = N_{max}$. These are two extrema which I use to substitute unknown constants in equation (3.2) and transform it to (3.3) using the following formula:

$$A \cdot \gamma \cdot (1 - k \cdot m) = 1/C; \qquad (3.4)$$

$$k \cdot \gamma/(1 - k \cdot m) = 1/N_{max}. \qquad (3.5)$$

When $N/N_{max} \sim 1$, the constrain factor becomes essential and the rate of population growth declines. The normalized relative population growth rate function

$$Y = 4(C/N) \cdot dN/dT = 4 \cdot (N/Nmax) \cdot (1 - N/Nmax) \qquad (3.6)$$

takes the form of an inverted square parabola (fig. 3.4).

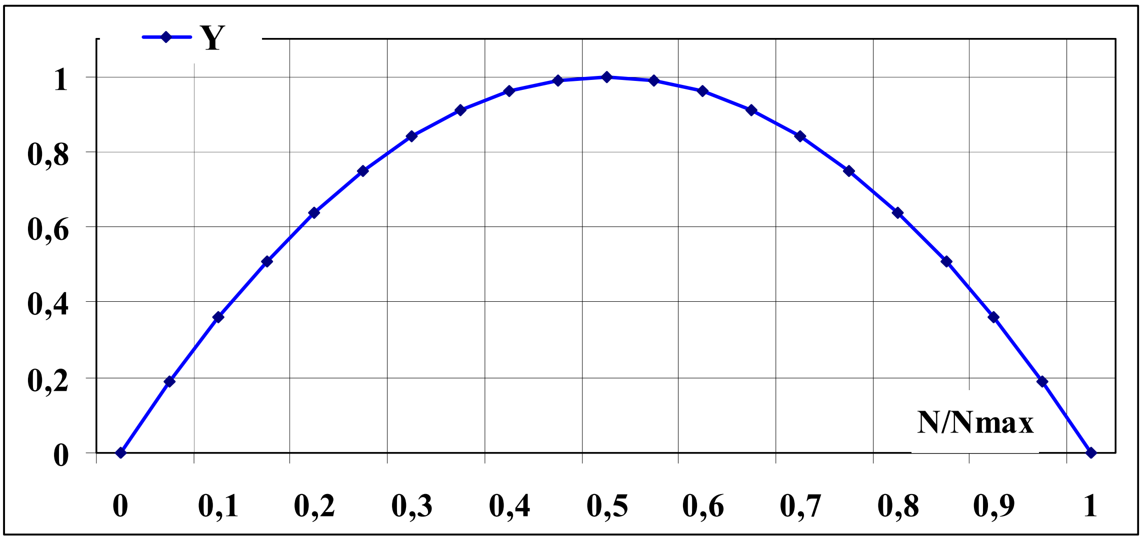

Figure 3.4. Normalized relative population growth rate function

It is worth noting that equation (3.3) may be tested for its adequacy directly. For example, when dN/dT is known numerically, it is easy to calculate the maximum global population

$$N_{max} = N/(1 - C(dN/dT)/N^2). \qquad (3.7)$$

In 1995, the global population growth rate was dN/dT = 87.4 mln people per annum, N = 5,682 mln people[89]. With C = 160 bln people per annum, N_{max} = 10 bln people approximates to the forecasted maximum population and proves the validity of equation (3.3).

3.2. Computational solution

Computational solution of the differential equation (3.3) is represented in figure 3.5 and indicated as "F2" (mln people). Compare it with the solution proposed by S.P. Kapitsa (F1) also represented in this figure. Here C is a constant from equations (1.2) and (3.3) the value of which C = 160 bln people per annum was chosen to achieve the best approximation, and N_{max} = 10,150 mln people.

Figure 3.5 shows the solution of this equation differs from S.P. Kapitsa's curve relatively inessentially. Its deviation from the statistics, similarly to Kapitsa's curve, is most visible in the beginning of the 20 century because of two world wars, Spanish influenza pandemic and civil war in Russia that account for up to a 10 percent deviation from the theoretic figures. After 1960, i.e. in the demographic transition period, the deviation from statistics lies within a 5 percent area and from F1 curve within 3.5 percent.

Table 3.1 provides a more precise comparison of different demographic transition equations. It also shows figures of global population **N** used by S.P. Kapitsa.[90]. Here **ΔN/N** is a relative deviation of the solution from the statistics. The figures prove the suggested solution F2 fits the statistics quite well and is even closer to S.P. Kapitsa's theoretical curve that also could not embrace such factors as wars and pandemics.

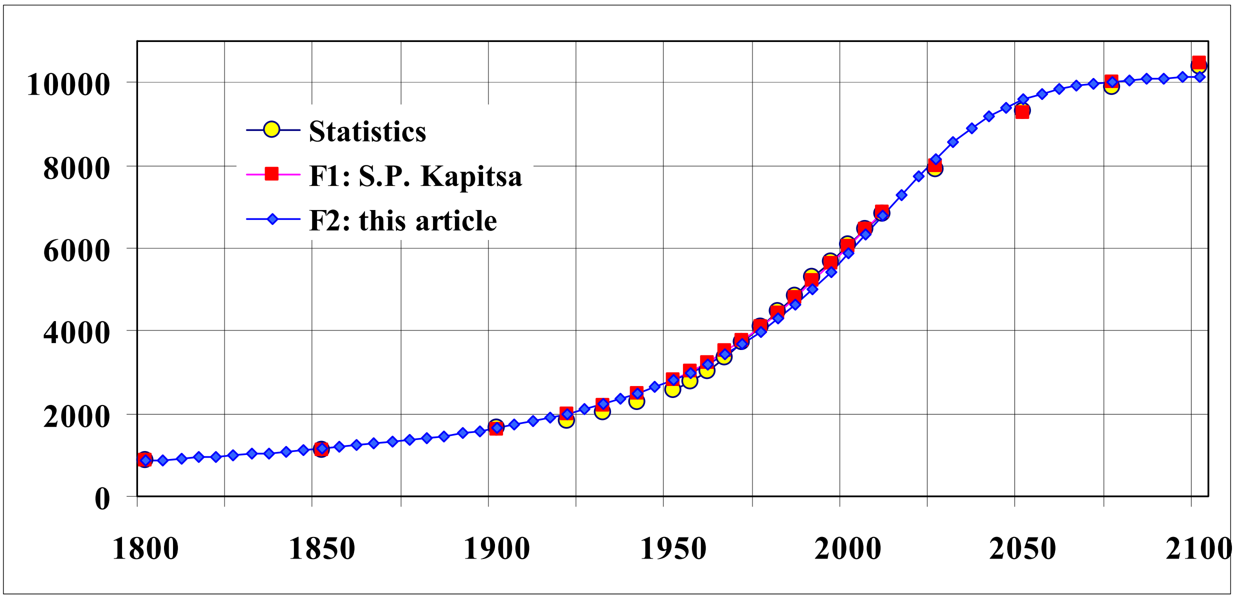

Figure 3.5. Versions of the demographic transition curves (mln people)

Note that since 1960 afterwards the world history witnessed the highest rate of economic growth, no serious wars and crises and rapid postcolonial economic growth in the Third World. That time the global population grew so rapidly that not only compensated for losses in the early 20 century but in 1975-2000 the actual figures even exceeded the theoretical numbers by 3 to 5 percent (see fig. 3.5).

[89] Капица С.П. Парадоксы роста: законы глобального развития человечества. – М., 2012.
[90] ibid.

Table 3.1. Demographic transition curve error

Period	Statistics N, mln	F1 N, mln	F2 N, mln	ΔN/N,% (F1)	ΔN/N,% (F2)
1800	870	851	865	–2.2	–0.6
1850	1,100	1,120	1,144	1.8	4.0
1900	1,660	1,625	1,657	–2.0	–0.1
1920	1,811	1,970	1,996	8.8	10.2
1920	2,020	2,196	2,215	8.7	9.7
1940	2,295	2,474	2,479	7.8	8.0
1950	2,556	2,817	2,800	10.2	9.6
1960	3,039	3,245	3,194	6.8	5.1
1970	3,707	3,778	3,681	1.9	–0.7
1980	4,454	4,430	4,281	–0.5	–3.9
1990	5,277	5,198	5,009	–1.5	–5.1
2000	6,073	6,038	5,863	–0.6	–3.5
2010	6,832	6,878	6,802	0.7	–0.4
2025	7,896	7,987	8,161	1.2	3.4
2050	9,298	9,259	9,576	–0.4	3.0
2075	9,879	9,999	10,021	1.2	1.4
2100	10,400	10,451	10,123	0.5	–2.7

3.3. Analytical solution

There is an analytical solution to the equation (3.3). To do so, let's introduce a dimensionless variable $X = N/N_{\mathbf{max}}$ and transform the equation (3.3) into

$$\mathbf{(1 / (X^2 \cdot (1 - X)) \cdot dX = (N_{max}/C) \cdot dT.} \qquad (3.8)$$

The solution of this equation is as follows

$$\mathbf{1/X - Ln\ (X/(1 - X)) = (N_{max}/C) \cdot (T_1 - T).} \qquad (3.9)$$

Noting the variable N, it is

$$\mathbf{T = T_1 - C/N - (C/N_{max}) \cdot Ln(N/(N_{max} - N)).} \qquad (3.10)$$

The parameter $C/N_{\mathbf{max}}$ has the dimension of time and represents the demographic transition time $C/N_{max} = N_0 \cdot T_0 / N_{max} \approx 16$ years. Characteristic demographic transition time $t_1 = C/N_{max} = N_0 \cdot T_0 / N_{max}$ allows to introduce a dimensionless time parameter $t = T/t_1 = T \cdot N_{max}/T_0 \cdot N_0$ to the equation (3.8) and so transform it to the fully dimensionless with no dimensionless similarity parameter. Hence the solution to the dimensionless equation takes the following general form

$$\mathbf{N/N_{max} = F(T \cdot N_{max}/T_0 \cdot N_0) = F(T/ t_1).}$$

It is interesting that T_1 is not a parameter in the computational solution; fixing a start point for the calculation, for example N(T=1800 year) or a point where one wants to achieve a good correlation of the results, would quite compensate for the absence of this parameter. The date T_1 acts as a sort of a reference point and if changed it shifts the whole curve along the time axis. The analytical solution correlates well with the statistics when $C/N_{\mathbf{max}} = 16$ years, $N_{\mathbf{max}} = 10…10.15$ bln people and $T_1 = 2022$ year. The analytical and computational solutions give the same N(T) dependence.

3.4. Analysis of the solution parameters

To perceive better the meaning of the obtained solution, let's turn back to the values of **A** and **k** constants in the equations (3.1), (3.2).

Coefficient k determines at what value of G/N people prefer employment to parenting (actually this represents the demographic transition). Dimension of k is [people per annum/dollar]. The above expression (3.5) for constant k gives $k = 1/\gamma \cdot N_{max} \cdot (1 + m/\gamma \cdot N_{max})$. Since $m/\gamma \cdot N_{max} \approx 0.02$ then $k = 1/\gamma \cdot N_{max}$ within 2 percent accuracy. If N_{max} =10,000 mln people, $1/k \approx$ 10,400 dollars per capita per annum in GK dollars as per 1995.

Coefficient A determines a relative rate of global population growth as a function of GDP per capita growth that correlates with accumulation of knowledge by mankind that in its turn depends on the number of people. Expression (3.4) gives $A = 1/C \cdot \gamma \cdot (1 - k \cdot m) = 1/C \cdot \gamma \cdot (1 - m/\gamma \cdot N_{max}) \approx 1/C \cdot \gamma$. If $C \approx 16 \cdot 10^{10}$ people per annum, $A \approx 1/(\gamma \cdot C) = 6 \cdot 10^{-6}$ people/dollar.

The key parameter in (3.10) is the ratio C/N_{max} that has the dimension of time. Express $N_{\mathbf{max}}$ and C in terms of constants A, **γ**, k and get $C \cdot N_{\mathbf{max}} = k/A = 16$ years. So this parameter represents a ratio of G/N at the demographic transition point to the G/N growth coefficient as a function of the global population. It means in fact how quickly the level of GDP when families change their demographic behaviour will be achieved.

Decline of fertility despite higher welfare becomes essential with the constraint factor $k \cdot G/N \sim 0.3$. At that $(G/N)_{\mathbf{dem}} \approx 0.3/k \approx \approx 3{,}120$ dollars per capita (here dollars mean GK as per 1995; in 2011 this variable is higher by about 37 percent and equals ~4,260 dollars). This variable achieved that value globally by 1960 and since that time the humanity has been witnessing rapid decline of fertility (see fig. 3.1). It is interesting that this value is about 15 times as much the cost of living that provides for a zero reproduction m.

The obtained equations (3.1), (3.3) show what characteristics of humanity as a synergy system affect the population growth and reaching the demographic transition point:

- in the initial stage of hyperbolic growth, it is a coefficient C that characterizes the growth population rate as a function of growth in the living standard (G/N) and the number of people;
- closer to the demographic transition point, it is a benchmark beyond which women prefer employment to parenting $(G/N)_{\mathbf{dem}} \approx$ 4,260 dollars per capita (dollar as per 2011) and the typical demographic transition time scale $t_1 \approx C \cdot N_{\mathbf{max}} \approx 16$ years.

Note also that satisfactory results available due to the type of the constraint factor adopted in the equation (3.1) (see fig. 3.4) depend also on the fact that there are both developed and underdeveloped countries in the world. Should humanity be more homogeneous the solutions would probably cover depopulation as is the case. The equation (3.1) if modified in some way seems to be applicable to a particular country as well but equations (3.3), (3.6) will be different in this case.

3.5. Systems effects

The demographic transition model considered above gives not only a lower fertility rate but also a higher amount of productive employees (women) and potential inventors. This factor should be investigated in more details later.

It is interesting to note that throughout its history humanity as a system developed in a rather unnatural way, namely its key parameter, the population number, provided a positive feedback. A nuclear explosion is an example of such a system. Until all the nuclei complete the reaction, the fission develops exponentially. Humanity, as it is mentioned above, grew by a no less rapid function, hyperbolic, so that the population increased from 100,000 to 7,000,000,000 people. And differently from the nuclear explosion, the number of ‘active agents’ here yet increases rather than disappears.

An exponent often acts as a byword for the most fast-growing function and a hyperbola differs from it in that a hyperbola grows much faster in its final stage and much slower at the beginning. That is why humanity developed quite slowly over a very long period and very rapidly afterwards.

Normally complex systems are elastic and restore their state once a deviation from equilibrium has occurred. They are surprisingly resistant to various effects. It is only effects on particular points may destabilize such systems and bring them to another state. Population density beyond the level of comfortable life, exhausted natural resources or dropped welfare (GDP per capita) might be taken as such points. However they ensure no evidences of any negative feedback in fact. ***It is a popular alternative to parenting that appears to be such a particular point of this system.***

Note another effect of complex systems. The hypothesis used to establish equation (3.1) that fertility decreases when $(G/N)_{\mathbf{dem}}$ achieves a definite level does not mean that the opposite is true. The hypothesis is based on the assumption that in the society of high GDP one type of population reproduction is replaced with another because employment, according to the alternative cost principle, appears more profitable. However this transition does not directly follow the moment when employment becomes more profitable but when profitability of this transition covers the cost incurred by the need of professional training, finding job, changing life style, arranging care of the family, etc.

However in societies where female employment is socially acceptable behaviour, the opposite transition depends on entirely different factors including social and cultural ones that essentially decrease the alternative cost of parenting. As a result the opposite transition does not, as a rule, occur even when the gross domestic product per capita drops dramatically, i.e. there is a hysteresis phenomenon.

Combine the equation (1.11) with the idea that maximum population N_{max} is limited by a definite figure and get another important conclusion. It means the maximum GDP per capita figure is limited

$$\mathbf{g_{Max} = (G/N)_{Max} = m + \gamma N_{max} \approx 10{,}621 \text{ dollars per capita per annum}} \qquad (3.11)$$

(in GK as per 1995 with $\mathbf{N_{max}}$ = 10 bln people). To compensate in some way for the equation (1.11) error and the need to adjust dollars as to the relative year, the equation (3.11) may be transformed as follows:

$$\mathbf{g_{max}/g \approx (m + \gamma N_{max})/(m + \gamma N).} \qquad (3.12)$$

Within the 2 percent accuracy, the equation (3.12) may be transformed as follows

$$\mathbf{g_{max}/g \approx N_{max}/N.} \qquad (3.13)$$

This implies that the global GDP per capita may increase just by 42 percent after 2011 and the global GDP may double in dollars as per 2011, i.e. may achieve up to $\mathbf{G_{Max}} \approx$ 180 bln dollars (if $\mathbf{N_{max}}$ > 10 bln people, then $\mathbf{G_{Max}}$ will be accordingly higher).

But according to PwC[25] forecast (see fig. 1.8), in 2050 GDP in PPP terms of 20 most developed economies will achieve 214 tln dollars as per 2011 (3.5 times as much) that corresponds to the global GDP of about 273 tln dollars. Thus this figure is less by about 1.6 times than PwC forecast.

Why there is a contradiction between these forecasts? On one hand, as it was mentioned above the PwC forecast misses the effect of cooperation between countries. On the other hand, the transition of humanity as a system to a new state due to the demographic transition may result in many characteristics of the system being changed including GDP being over the figure derived from the equation (3.1).

There is a definite background for this. In particular, note that the rapid growth of GDP in E7 countries follows the rapid growth of productivity (G/N) in the group of countries with much larger population (4.5 times) than in G7 countries. And the growth of productivity depends on the technology diffusion from G7 to E7 countries since E7 countries are far behind with respect to developing new technologies.

However as far as 100 years ago emerging countries did not develop as fast by means of developed countries. Starting from the last century technology has been transferred from one countries to the others in an evidently different way. And this process may change further even more significantly. On one hand, barriers for technology transfer may become lower (if confrontation does not slow down this process). On the other hand, emerging economies on their own may start contributing much more to developing technology and this would be beneficial for the developed countries as well due to the synergy effect. Reliability of the forecasts may heavily depend on the way a certain alternative is implemented.

Note another consequence of the above forecast about possible cessation of growth of GDP and GDP per capita. Stagnation of human development is one of the after-effects. Modern business is aimed at ever growing production which if lacked is considered as a great problem. The above result means the growth may stop totally and only fluctuations or slow drifting will remain. This implies just an entirely different way of economy being rather than a permanent crisis. Moreover, innovation processes may take an entirely different nature – no principally new technology will emerge but the old will be implemented at new places and under new conditions. This conclusion results from the transition of humanity to the demographic transition stage and further to 'demographic stabilization' though there are alternatives that will be considered below.

From the perspective of the desire to increase global life standards, it is essential what population control strategy will be followed. Many authors starting from Malthus believe the global population growth should be constrained and so even now discuss theories of 'The Golden Billion' type. And some countries are implementing programmes on population control.

Meanwhile the above analysis (equation 3.13) proves the increase of the ultimate global population $\mathbf{N_{max}}$ may promote welfare globally including mass population of rich countries. However to increase global population and compensate for depopulation of developed countries is attainable by means of ethnic groups of low GDP per capita and correspondingly high fertility (see fig. 3.3). So global community should be very careful about possible growth of global population and consider it as a *potentially best demographic strategy*. The danger of exhausting natural resources needs careful consideration as well.

Key results of chapter 3

1. Differential equation of growth of global population (N) as a function of time (T) is as follows

$$\mathbf{dN/\,dT = (1/C)\cdot N^2 \cdot (1 - N/N_{max})},$$

and its analytical solution fitting well the statistics is

$$\mathbf{T = T_1 - C/N - (C/N_{max})\cdot Ln(N/(N_{max} - N))}.$$

Despite the fact that the solution to the equation (3.3) is based on a phenomenological expression for G/N (1/11), it has a more fundamental meaning in (3.1) and demonstrates that the key factor influencing the growth of population is the gross domestic product per capita, i.e. the economic factor rather than proximity to the date of *singularity* as it is in the equation suggested by S.P. Kapitsa.

The considered mathematical model of growth of population has a range of advantages over other authors' models. For example, differently from M. Kremer's model, the considered model gives the result in the form of analytical functions without any extra empiric parameters inserted and this makes the model clearer and easier to test its adequacy. Differently from A.V. Korotaev, A.S. Malkov, D.A. Khalturina's model[91], the considered model does not require to introduce a variable of female literacy that is essential from Occam's razor perspective.

The suggested approach to solving demographic transition problems indicates how important for population dynamics the act of decision-making by families on two alternatives is: whether to parent or be employed. Though GDP per capita dominates this choice, developed countries are potentially able to provide resources to motivate families alternatively in order to ensure the country its demographic independence.

Important are the conclusion that GDP and GDP per capita may stop growing and the conclusion that humanity as a system may enter the stage of absence of growth (stagnation).

[91] Коротаев А.В., Малков А.С., Халтурина Д.А. Математическая модель роста населения Земли, экономики, технологии и образования. – М., 2005.